XLiFE++: a FEM/BEM multipurpose library

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eXtended Library of Finite Elements in C++

Born in 2011 of a tradition of computational softwares since the end of 80s in P.O.E.M.S.
Development team split into P.O.E.M.S. (Nicolas Kielbasiewicz, Eric Lunéville, Colin Chambeyron and Nicolas Salles) and IRMAR (Yvon Lafranche, Pierre Navaro and Eric Darrigrand)

3 first years funded by european project SIMPOSIUM
3 last years funded by DGA
Web site: http://uma.ensta-paristech.fr/soft/XLiFE++/

Tutorial: Helmholtz with periodic conditions

```
#include "xlife++.h"
using namespace xlifepp;

Real uexa(const Point& P, Parameters& pa = defaultParameters)
{ return cos(2.*pi_*P.x)*sin(2.*pi_*P.y); }

Real f(const Point& P, Parameters& pa = defaultParameters)
{ return (8.*pi_*pi_-1.)*uexa(P,pa); }

Vector<Real> mapPM(const Point& P, Parameters& pa = defaultParameters)
{ Vector<Real> Q=P; Q(1)-=1;
  return Q; }

int main(int argc, char** argv)
{
  init(_lang=en); // mandatory initialization of xlifepp
  Strings sn("Gamma","sigmaP","Gamma","SigmaM");
  Square sq(_center=Point(0.,0.), _length=1, _nnodes=Numbers(21, 41, 21, 21), _domain_name="Omega", _side_names=sn);
  Disk d(_center=Point(0.,0.), _radius=0.2, _nnodes=8, _side_names="Gamma");
  Mesh mesh2d(sq-d, triangle, 1, gmsh);
  Domain omega = mesh2d.domain("Omega"), gamma = mesh2d.domain("Gamma");
  Domain sigmaM=mesh2d.domain("x=0"), sigmaP=mesh2d.domain("x=1");
  defineMap(sigmaP, sigmaM, mapPM);  //useful to periodic condition

  Space Vk(omega, P1, "Vk");
  Unknown u(Vk, "u");
  TestFunction v(u, "v");

  BilinearForm auv = intg(omega, grad(u) | grad(v)) - intg(omega, u * v);
  LinearForm fv=intg(omega, f * v);
  EssentialConditions ecs = (u|gamma=uexa) & ((u|sigmaP) - (u|sigmaM) = 0);

  TermMatrix A(auv, ecs, "a(u,v)");
  TermVector B(fv, "f(v)");

  TermVector U = directSolve(A,B);
  saveToFile("U", U, vtu);
}
```

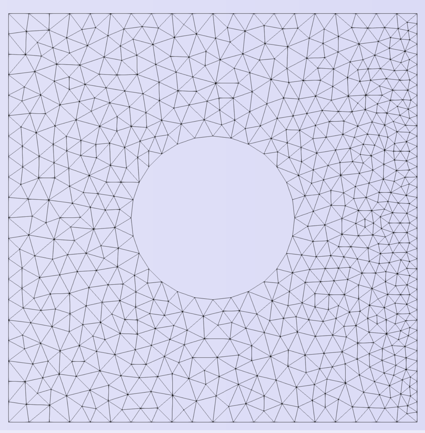
uexa and f are user functions used in the variational problem.
mapMP is the domain mapping used for periodic essential conditions.

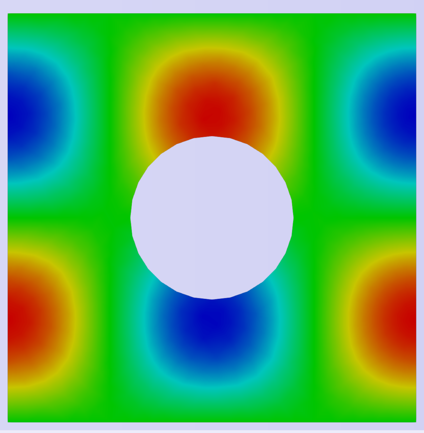
Definition of geometrical data (mesh, domains, ...)
⟶ wrapper to GMSH mesh generator

Definition of approximation space, unknown and test function

Definition of variational problem

Assembly of linear system, solve and export to PARAVIEW VTU format here





Features

- Deals with 1D / 2D / 3D problems
- Deals with scalar / vector problems
- Deals with transient / stationnary / harmonic problems
- Deals with single / multiple unknown(s) problems
- Various approximation methods:
  - Lagrange finite elements *at any order*
  - 2D / 3D H_{rot} and H_{div} finite elements *at any order*
  - Spectral elements
- Various numerical methods ***that can be naturally coupled***:
  - 1D / 2D / 3D / Finite element method
  - 2D / 3D boundary integral equations methods + integral representation: Laplace, Helmholtz and Maxwell kernels
  - ***Absorbing conditions***: DtN, PML, ...
- ***Any set of linear essential conditions***: Dirichlet, transmission, periodic, quasi-periodic, ...
- Many internal solvers (direct solvers, iterative solvers, eigenvalue solvers) and wrappers to ARPACK, UMFPACK, MAGMA libraries
- Parallel programming : multithreading with OpenMP, GPGPU through MAGMA library

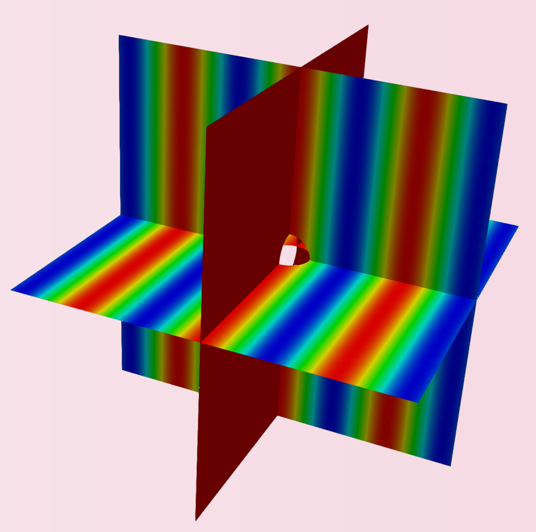
Roadmap

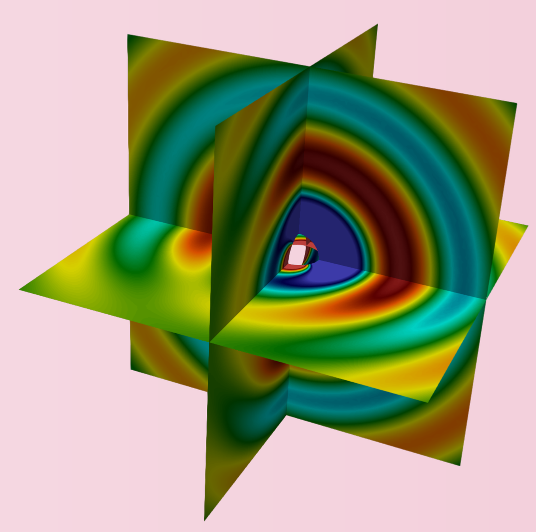
- Galerkin discontinuous methods
- Fast multipole methods (coupling with FASTMMLIB, developed by Eric Darrigrand, I.R.M.A.R.)
- Domain decomposition methods

XLiFE++

Electromagnetic wave scattering by a sphere (EFIE)

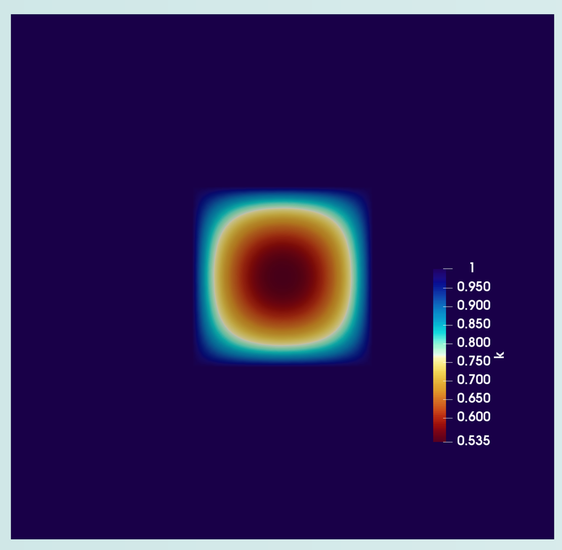
This example shows results of a 3D Maxwell problem using EFIE with homogeneous Dirichlet boundary condition (i.e. $\mathbf{E} \times n = 0$) where the scatterer is the unit sphere centered at the origin and the incident electric field is the plane wave $\mathbf{E} = e^{ikz}\mathbf{e_1}$
EFIE formulation uses Raviart-Thomas elements in order to compute the electric current along the sphere scatterer. We represent here the first component of the incident electric field (on the left) and of the scattered electric field (on the right) on planes x=0, y=0 and z=0.

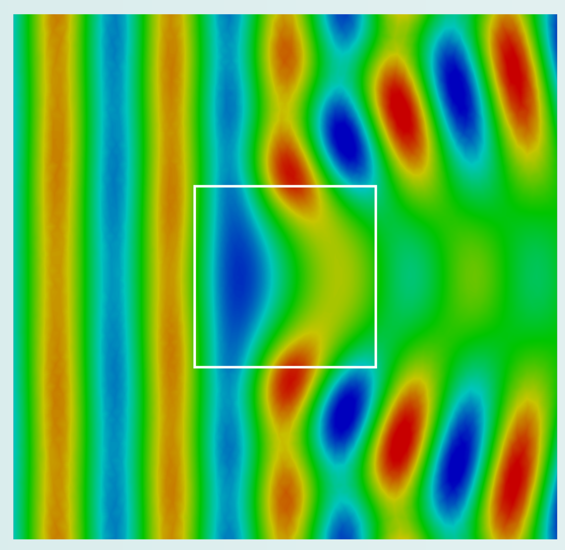




Acoustic wave scattering by FEM-BEM coupling

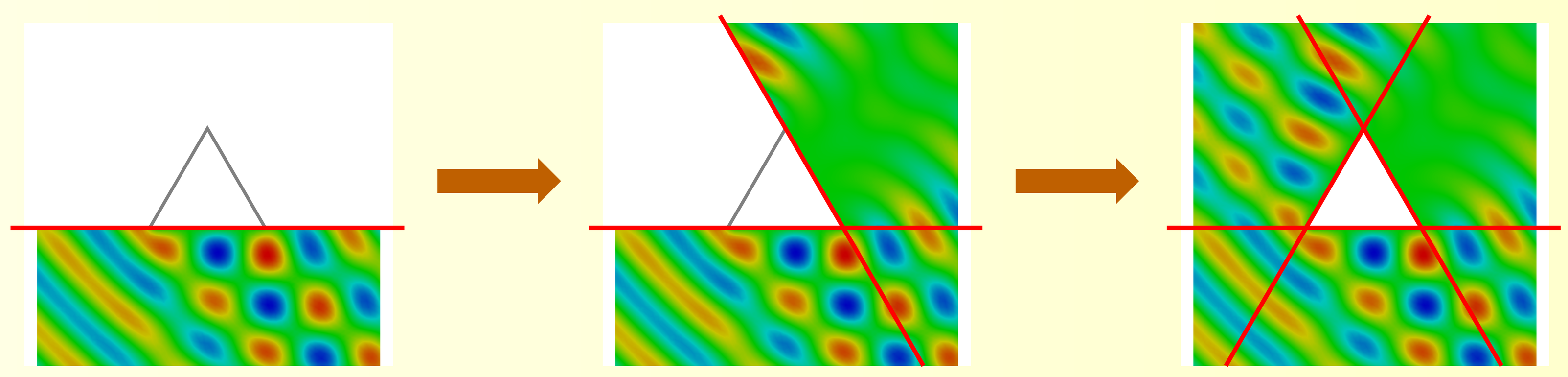
FEM-BEM coupling is used to solve wave scattering problems in infinite homogeneous media with a bounded heterogeneous part. Are displayed on the left the wave number profile and on the right, the total field. The boundary between FEM part and BEM part is highlighted. As far as BEM is concerned here, both single layer and double layer potentials are used.





Wave scattering by Half-Space Matching Method

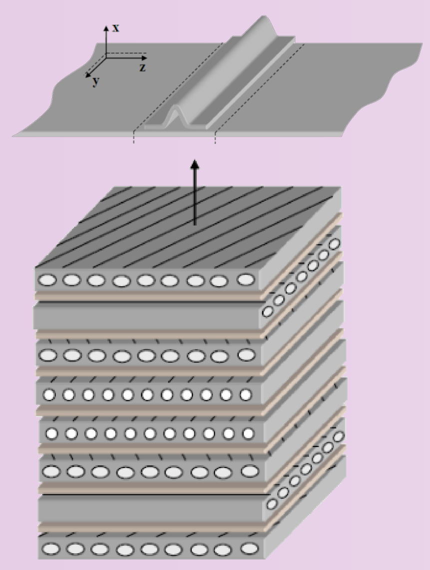
The acoustic scattering problem is rewritten as a system of coupled integral equations whose unknowns live on the lines (shown in red in the following figures) supported by the edges of the polygonal scatterer. Reconstruction on each half-space uses truncated Fourier transform. (See [Bon+17])



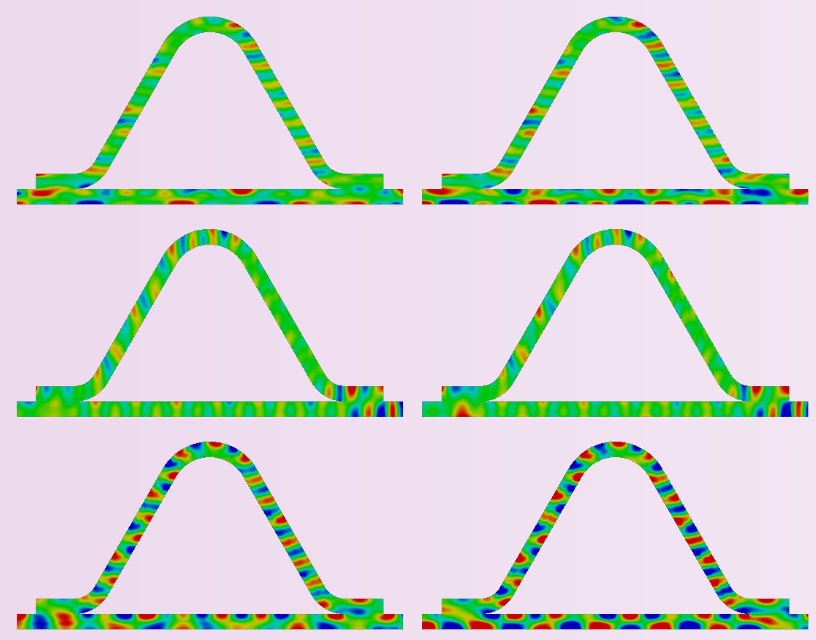
[Bon+17] Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Yohanes Tjandrawidjaja, and Antoine Tonnoir. "The Half-Space Matching Method for the diffraction by polygonal scatterers". In: WAVES 2017. 2017.

Elastic wave propagation in a plane stiffener

The computation of wave propagation in a multilayer anisotropic elastic waveguide is based on numerical hybrid DtN (See [Bar09]). To build them, eigenvectors in 1D cross section of the stiffener are computed and then used in tensor kernels involved in DtN operators.



Two first modes:



[Bar09] Vahan Baronian. "Couplage des méthodes modale et éléments finis pour la diffraction des ondes élastiques guidées. Application au Contrôle Non Destructif". PhD thesis. Nov. 2009.